## Homework Assignment 5 – 600.445 Fall 2004

# Instructions and Score Sheet (hand in with answers)

| | |
|---|---|
| Name | |
| Email | |
| Other contact information (optional) | |
| Signature (required) | I have followed the rules in completing this assignment<br><br>_____ |

| Question | Points | Points | Total |
|---|---|---|---|
| 1 | | 20 | |
| 2 | | 10 | |
| 3 | | 50 | |
| 4 | | 20 | |
| Total | | 100 | |

1. Remember that this is a graded homework assignment. It is the functional equivalent of a take-home exam.
2. You are to work **alone** and are **not to discuss the problems with anyone** other than the TAs or the instructor.
3. It is otherwise open book, notes, and web. But you should cite any references you consult.
4. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
5. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
6. Sign and hand in the score sheet as the first sheet of your assignment.
7. Remember to include a sealable 8 ½ by 11 inch self-addressed envelope if you want your assignment

# Scenario

The scenario for this problem is stereotactic localization of a target in biplane x-ray images. Assume that we have the following information:

$\vec{\mathbf{b}}$ = (unknown) position of a point in patient coordinate system

$\mathbf{F}_1$ = nominal detector pose (wrt patient coord sys.) when image 1 is taken

$\mathbf{F}_2$ = nominal detector pose (wrt patient coord sys.) when image 2 is taken

$\vec{\mathbf{s}}$ = nominal source position (wrt detector)

$\vec{\mathbf{d}}_1$ = nominal position (in detector coordinates) of image of $\vec{\mathbf{b}}$ in image 1

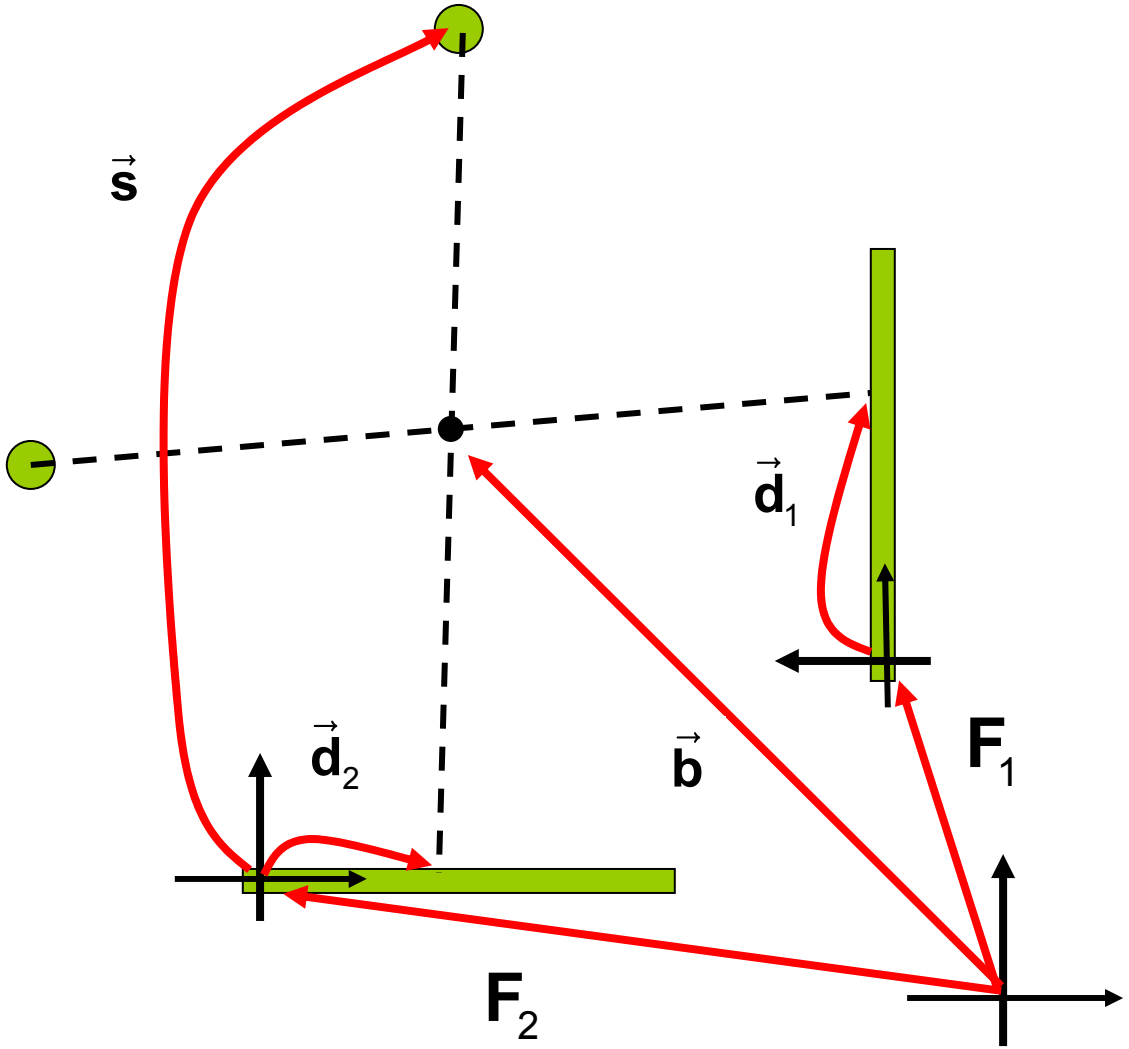
$\vec{\mathbf{d}}_2$ = nominal position (in detector coordinates) of image of $\vec{\mathbf{b}}$ in image 2

Further, we know that the source is positioned roughly over the midpoint of the detector. The two images are taken with the target point $\vec{\mathbf{b}}$ roughly half way between the source and detector in both images, and the images are taken from viewing directions that are roughly at right angles to each other. In fact, you can assume that nominally the detector corners are at [0,0] and [200,200] and

$$\vec{\mathbf{s}} = [100,100,1000]$$

$$\mathbf{F}_1 \approx \left[Rot(\vec{\mathbf{x}},90^\circ),[1000,1600,1400]\right]$$

$$\mathbf{F}_2 \approx \left[\mathbf{I},[1000,1000,1000]\right]$$


$\vec{s}$
$\vec{d}_1$
$\vec{d}_2$
$\vec{b}$
$F_1$
$F_2$

# Questions

1. (20 points) Suppose that we know that there is some tracking error, so that the actual poses of $\mathbf{F}_1$ and $\mathbf{F}_2$ are given by

$$\mathbf{F}_1{}^* = \mathbf{F}_1 \bullet \Delta\mathbf{F}_1 = \mathbf{F}_1 \bullet \left[ Rot(\vec{\boldsymbol{\alpha}}_1), \vec{\boldsymbol{\varepsilon}}_1 \right]$$

$$\mathbf{F}_2{}^* = \mathbf{F}_2 \bullet \Delta\mathbf{F}_2 = \mathbf{F}_2 \bullet \left[ Rot(\vec{\boldsymbol{\alpha}}_2), \vec{\boldsymbol{\varepsilon}}_2 \right]$$

where $\vec{\boldsymbol{\alpha}}_k$ and $\vec{\boldsymbol{\varepsilon}}_k$ are small angles and displacements. Give approximate expressions for:

$\vec{\mathbf{s}}_{P1}{}^* =$ actual position of the source wrt the patient for image 1

$\vec{\mathbf{s}}_{P2}{}^* =$ actual position of the source wrt the patient for image 2

$\vec{\mathbf{d}}_{P1}{}^* =$ actual position of the target image point wrt the patient for image 1

$\vec{\mathbf{d}}_{P2}{}^* =$ actual position of the target image point wrt the patient for image 2

in terms of $\mathbf{R}_k$, $\vec{\mathbf{p}}_k$, $\vec{\boldsymbol{\alpha}}_k$, etc. (Hint: the nominal positions involve expressions like $\vec{\mathbf{d}}_{Pk} = \mathbf{F}_k \bullet \vec{\mathbf{d}}_k$).

2. (10 points) Now give expressions for these quantities assuming that the nominal values are those given in the preliminary description.

3. (50 points) Assuming that $\vec{\mathbf{b}}^*$ is roughly [1100,1100,1500] in patient coordinates, and that a triangulation method similar to that outlined in Programming Assignment 2 is used to produce an estimate $\vec{\mathbf{b}}$ of the target point position in patient coordinates, give an expression suitable for estimating the error

$$\Delta\vec{\mathbf{b}} = \vec{\mathbf{b}}^* - \vec{\mathbf{b}}$$

4. (20 points) Assuming that we know that

$$\left|\alpha_{k,x}\right| \le 1^\circ \quad \left|\alpha_{k,y}\right| \le 1^\circ \quad \left|\alpha_{k,z}\right| \le 1^\circ$$

$$\left|\varepsilon_{k,x}\right| \le 1 \quad \left|\varepsilon_{k,y}\right| \le 1 \quad \left|\varepsilon_{k,y}\right| \le 1$$

estimate bounds on $\Delta\vec{\mathbf{b}}$.