Homework Assignment 5 – 600.445 Fall 2004

Instructions and Score Sheet (hand in with answers)

Name	Name
Email	Email
Other contact information (optional)	Other contact information (optional)
I/we have followed the rules in completing this assignment ________________________________ Signature (required)	I/we have followed the rules in completing this assignment ________________________________ Signature (required)

Question	Points	Points
1A	5	
1B	10	
1C	5	
2A	10	
2B	15	
2C	15	
2D	15	
2E	5	
3A	5	
3B	5	
3C	5	
4A	10	
4B	15	
Total	120	

1. Remember that this is a graded homework assignment. It is the functional equivalent of a take-home exam.
2. Because it is a design exercise, you may work in teams of **no more than two people.**
3. Aside from you partner, you are to work **alone** and are **not to discuss the problems with anyone** other than the TAs or the instructor.
4. It is otherwise open book, notes, and web. But you should cite any references you consult.
5. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
6. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
7. Sign and hand in the score sheet as the first sheet of your assignment.
8. Remember to include a sealable 8 ½ by 11 inch self-addressed envelope if you want your assignment returned.
9. Remember that there is no single "right" answer to a design exercise. Nevertheless, I am looking for sufficient analysis to justify your design choices.

Problem Scenario: “virtual fluoroscopy”

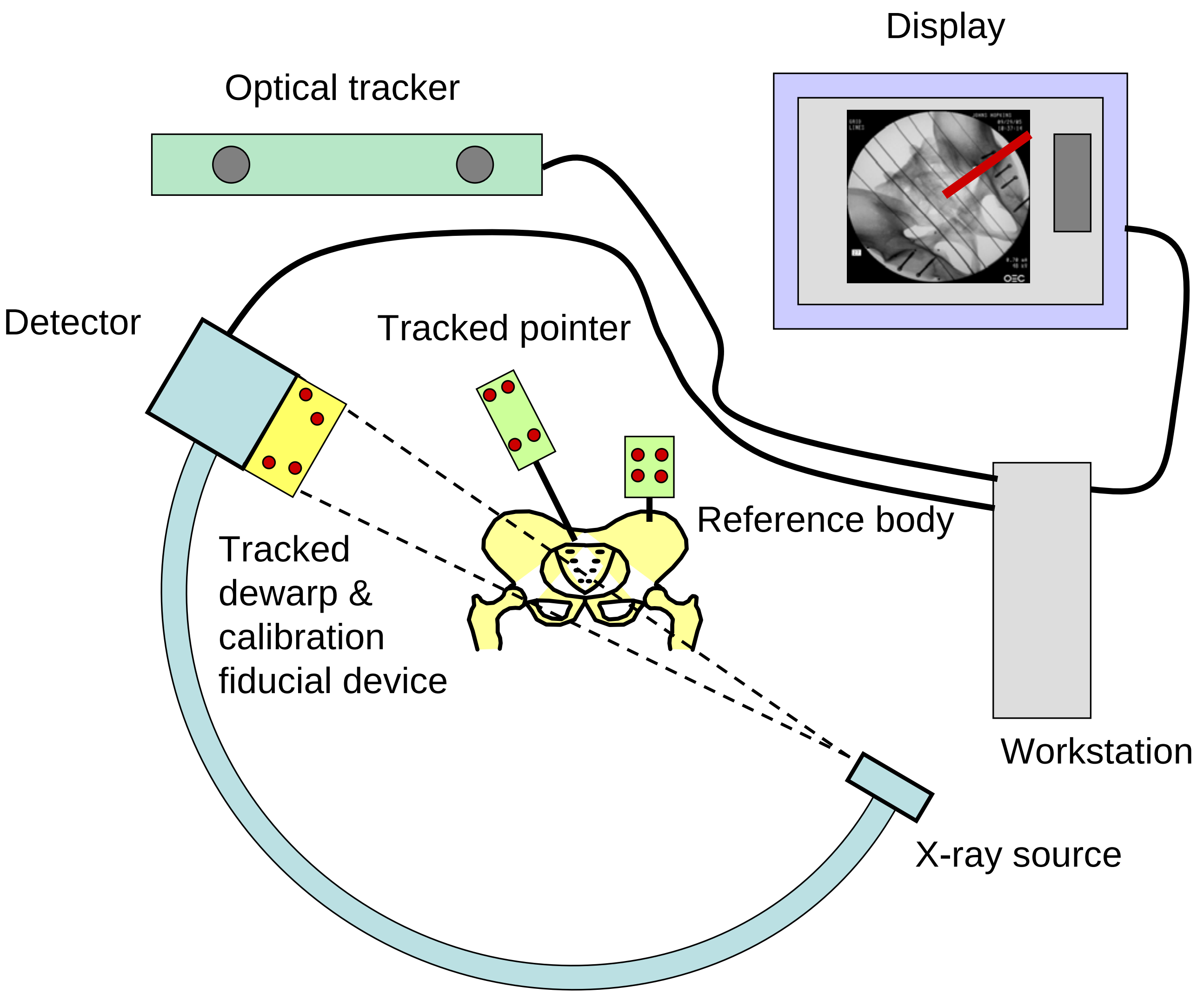

Figure 1

Consider the "virtual fluoroscopy" system shown in Figure 1. This system consists of the following elements:

- An optical tracking system capable of locating optical markers with an absolute accuracy of 0.1 mm.
- A surgical pointer equipped with optical markers, whose pattern you are to design. The pointer shaft must extend at least 100 mm beyond any markers or structure designed to hold the markers.
- A surgical reference body equipped with optical markers and an adjustable clamping mechanism for attachment to the patient's anatomy. In normal use, the nearest part of the marker pattern (which you are to design) is approximately 100 mm from the tip of the pointer.
- A fluoroscopic x-ray c-arm system with a digital interface. The x-ray source is 100 cm from the x-ray detector of this system. The detector (unusually) is 256 mm square, and the images produced are 512 x 512 pixels. The images are subject to unknown distortions that vary with the position of the c-arm.
- A dewarping & calibration device (which you are to design) mounted on the c-arm detector. This device also has a number of optical markers attached to it at locations to be determined in the design phase.
- A computer workstation with image processing software and a graphical display.

The following information is known about the distortion function $\vec{\mathbf{h}}(u,v)$.

$$\vec{\mathbf{u}}_{actual} = [u_{actual}, v_{actual}] = \text{actual pixel location}$$

$$\vec{\mathbf{u}}_{observed} = [u_{observed}, v_{observed}] = \text{observed pixel location}$$

$$= \vec{\mathbf{u}}_{actual} + \vec{\mathbf{h}}(u_{actual}, v_{actual})$$

$$\vec{\mathbf{h}}(\vec{\mathbf{u}}) = \left[f(u,v), g(u,v) \right]$$

$$\left| f(u,v) \right| \le 20 pixels \quad \left| g(u,v) \right| \le 20 pixels$$

$$\left| \frac{\partial f}{\partial u} \right| \le 0.02 \qquad \left| \frac{\partial g}{\partial u} \right| \le 0.02$$

$$\left| \frac{\partial f}{\partial v} \right| \le 0.02 \qquad \left| \frac{\partial g}{\partial v} \right| \le 0.02$$

In other words, the largest distortion in any direction is at most 20 pixels and the rate of change in distortion is at most 2%.

The workstation's image processing software is capable of locating the centroids of circular blobs of various diameters. For blobs larger than 1 pixel in diameter, the accuracy is $1/\sqrt{d}$ pixels, where d is the blob diameter in pixels. (This is not entirely realistic, but, hey, it is a homework assignment). The software can also locate the images of line-like features to an accuracy of

$1/\sqrt{d}$ pixels in the direction perpendicular to the line, where d is the width of the line.

We will adopt the following conventions for coordinate systems:

$$\mathbf{F}_{\det}(t) = \text{pose (wrt tracker) associated with detector at time } t$$
$$\mathbf{F}_{ref}(t) = \text{pose associated with reference body at time } t$$
$$\mathbf{F}_{ptr}(t) = \text{pose associated with pointer body at time } t$$
$$\vec{\mathbf{p}}_{tip} = \text{position of pointer tip with respect to } \mathbf{F}_{ptr}(t)$$
$$\vec{\mathbf{s}}(t) = \text{position of xray source with respect to } \mathbf{F}_{\det}(t)$$
$$\approx \left[0,0,1000 \text{ mm}\right]$$

The basic function of the system may be outlined as follows:

Acquire image (time = t_{cap}): Use the tracking system to determine the values of $\mathbf{F}_{\det}(t_{cap})$ and $\mathbf{F}_{\text{ref}}(t_{cap})$. Acquire an image $\mathbf{Im}(t_{cap})$**.** Perform whatever processing is needed to compute the distortion function $\vec{\mathbf{h}}_{cap}(u,v)$ and accurately calculate the source position $\vec{\mathbf{s}}\left(t_{cap}\right)$.

Navigation (time = $t_{nav} \geq t_{cap}$): Use the tracking system to determine the values of $\mathbf{F}_{\text{ref}}\left(t_{nav}\right)$ and $\mathbf{F}_{\text{ptr}}\left(t_{nav}\right)$. From these compute the position $\vec{\mathbf{q}}\left(t_{nav}\right)$ of the pointer tip with respect to the image capture coordinate system.

$$\vec{\mathbf{q}}\left(t_{\text{nav}}\right)=\mathbf{F}_{\text{det}}^{-1}\left(t_{cap}\right)\bullet\mathbf{F}_{\text{ref}}\left(t_{cap}\right)\bullet\mathbf{F}_{\text{ref}}^{-1}\left(t_{nav}\right)\bullet\mathbf{F}_{\text{ptr}}\left(t_{\text{nav}}\right)\bullet\vec{\mathbf{p}}_{\text{tip}}$$

Use the values of $\vec{\mathbf{q}}\left(t_{nav}\right)$, $\vec{\mathbf{h}}_{cap}(u,v)$, and $\vec{\mathbf{s}}\left(t_{cap}\right)$ to compute the image coordinates $\vec{\mathbf{u}}_{\text{proj}}=\left[u_{proj},v_{proj}\right]$ corresponding to $\vec{\mathbf{q}}\left(t_{nav}\right)$. Display suitable graphics overlaid on a display of the image $\mathbf{Im}(t_{cap})$.

The following questions concern key elements of the design of this system.

Question 1: Pointer and reference body design

A. Sketch designs for the pointer and for the portion of the reference body containing the optical tracker markers. Your design should be suitable for locating the pointer tip with respect to the reference body to an accuracy of 0.4 mm, and it should include:

- The number and position of optical markers with respect to $\mathbf{F}_{ptr}$ and $\mathbf{F}_{ref}$, respectively.
- The value of $\vec{\mathbf{p}}_{tip}$
- Any other dimensions or structural details that you think appropriate.

Your answer should include a few sketches as well as the 3D coordinates of the optical markers.

B. Present analysis justifying the assertion that your design meets the accuracy requirements outlined above.

C. Discuss your design choices from the standpoint of any trade-offs involving ergonomics, accuracy, ease of manufacturability, cost, etc. that seem important.

Question 2: Xray dewarp/calibration fiducial structure design

A. Sketch the design of an x-ray dewarp/calibration fiducial structure suitable for determining the value of $\vec{\mathbf{h}}(u,v)$ to 1 pixel and the position of $\vec{\mathbf{s}}$ to within 5 mm. Your design should include:

- A simple sketch or sketches
- The number, design, and position of any fiducial structures.
- Values for any other important physical dimensions
- Symbolic names for any important positions (e.g., the positions for fiducial marks) for use in subsequent parts of this question.
- Sufficient text explanation to make your design clear.

Hint: Most designs will probably involve a radiolucent plate with some fiducial objects on it placed right on top of the detector and another radiolucent plate or other structure with fiducials placed somewhere above it. But there are many alternatives.

B. Explain with formulas and text how you will compute $\vec{\mathbf{h}}(u,v)$ and $\vec{\mathbf{s}}$.

C. Provide analysis showing how your design ensures that your estimate of $\vec{\mathbf{h}}(u,v)$ is accurate to 1 pixel.

D. Provide analysis showing how your design ensures that your estimate of $\vec{\mathbf{s}}$ is accurate to 5 mm. (**Note:** If you like, you can assume that your design has indeed guaranteed that your estimate of $\vec{\mathbf{h}}(u,v)$ is accurate to 1 pixel).

E. Discuss your design choices from the standpoint of any trade-offs involving ergonomics, accuracy, ease of manufacturability, cost, etc. that seem important.

Question 3: Fiducial Structure Tracking

A. Sketch a suitable design for the tracking portion of the x-ray fiducial structure. The goal should be to enable you to determine the position and orientation of $\mathbf{F}_{\text{det}}$ to 0.1 mm and 0.002 radians. I.e., if

$$\Delta\mathbf{F}_{\text{det}} \approx \left[\mathbf{I} + sk(\vec{\alpha}_{\text{det}}), \vec{\varepsilon}_{\text{det}}\right]$$

then

$$\left|\vec{\alpha}_{\text{det}}\right| \leq 0.002 \quad \text{and} \quad \left|\vec{\varepsilon}_{\text{det}}\right| \leq 0.1\,\text{mm}$$

Here, I am interested in the number and position of the optical markers with respect to $\mathbf{F}_{\text{det}}$.

B. Present analysis justifying the assertion that your design meets the accuracy requirements outlined above.

C. Discuss your design choices from the standpoint of any trade-offs involving ergonomics, accuracy, ease of manufacturability, cost, etc. that seem important.

Question 4: Navigation

A. Provide formulas for computing image coordinates $\vec{\mathbf{u}}_{\mathrm{proj}} = \left[u_{proj}, v_{proj} \right]$ corresponding to $\vec{\mathbf{q}}\left(t_{nav}\right)$.

B. Assuming that the pointer tip is approximately at position $\left[50, 50, 500\right]^T$ with respect to $\mathbf{F}_{\mathrm{det}}^{*}\left(t_{\mathrm{nav}}\right)$, and assuming that you have indeed estimated $\vec{\mathbf{h}}\left(u, v\right)$ to within 1 pixel and $\vec{\mathbf{s}}$ to within 5 mm, and that your fiducial tracking design also meets the requirements of Question 3, estimate the accuracy (in pixels) of your estimate for $\vec{\mathbf{u}}_{\mathrm{proj}}$.