

## Homework Assignment 4 – 600.445 Fall 2010

### Instructions and Score Sheet (hand in with answers)

Name	
Email	
Other contact information (optional)	
Signature (required)	I have followed the rules in completing this assignment _____

Name	
Email	
Other contact information (optional)	
Signature (required)	I have followed the rules in completing this assignment _____

Question	Points	Total	Points	Total
1A	5			
1B	10			
1C	20			
1D	15			
2	50			
<b>Total</b>		<b>100</b>		

1. Remember that this is a **graded** homework assignment. It is effectively an exam.
2. You are to work alone or in a team of two people and are not to discuss the problems with anyone other than the TAs or the instructor.
3. Put your names and email address on each sheet and number the sheets.
4. You are encouraged to make free use of any published materials, the web, etc. in developing your answer but a) you must give full and proper citations to any references consulted and b) you may not consult, discuss, or otherwise communicate about this assignment with any human being except your lab partner, the course instructor, or the TAs. The one exception is that you should not refer to previous years' homework.
5. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
6. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
7. Sign and hand in the score sheet as the first sheet of your assignment
8. Remember to include a sealable 8 ½ by 11 inch self-addressed envelope if you want your assignment.
9. Attach the grade sheet as the first sheet and attach all sheets together.
10. You must include a self-addressed, seal-able 8 ½ x 11 inch envelope if you expect to the homework to be returned (per JHU's interpretation of FERPA).

## Question 1

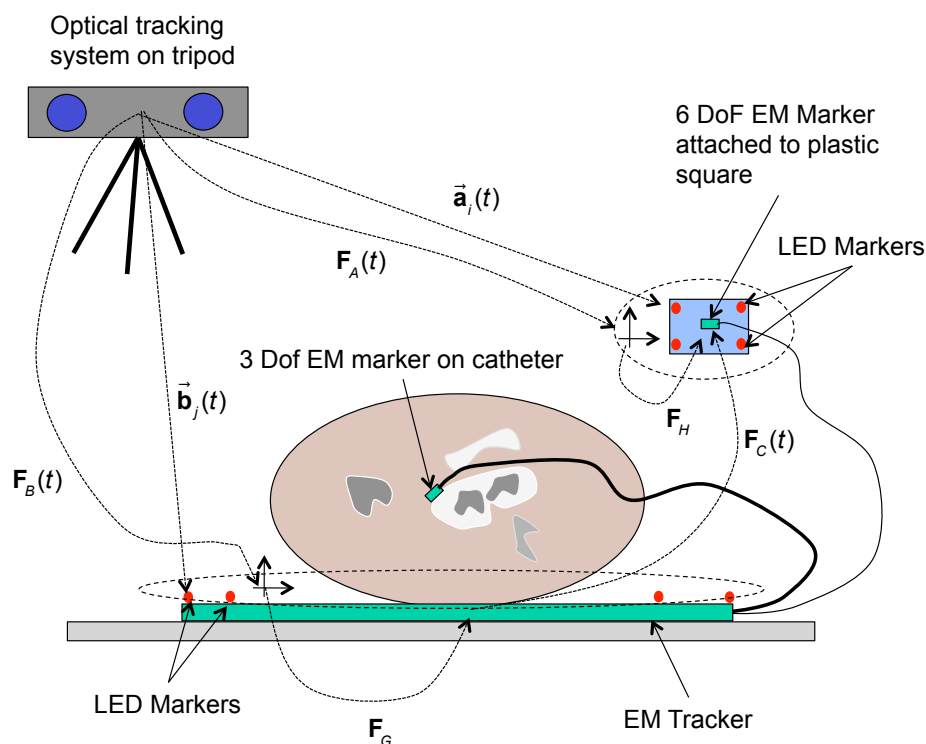


Figure 1

Consider the EM Tracker scenario shown in Figure 1, above. The “base unit” of the tracker is flat and sits under the patient. The tracking device tracks EM “markers”. One of these markers has been placed at the end of a catheter and will be threaded into the patient’s body. We wish to know the location of this catheter relative to the tracker base unit to an accuracy of 2 mm. Although reasonably accurate, the tracker system is nevertheless subject both to random errors of approximately 0.2 mm and rather larger systematic distortion errors. Thus, if a 3 DoF EM marker is at position  $\vec{v}_{actual}$  in EM tracker coordinates, the measured position will be

$$\vec{v}_{meas} = \vec{v}_{actual} + \vec{n} + \vec{d}(\vec{v}_{actual})$$

where  $\|\vec{n}\| \leq 0.2$  mm represents the random error and  $\vec{d}(\vec{v}_{actual})$  represents the distortion.

Although the actual distortion function is not known, we do know that it is small near the EM tracker and gets larger as the distance increases. For the purposes of this problem, we may assume that  $\|\vec{d}([x,y,z])\| \leq 0.0001 z^2$  where  $z$  is the distance of the EM marker from the top surface of the EM tracker base unit. We also know that the distortion is “smooth” and that

$$\left| \frac{\partial d_i}{\partial v_j} \right| \leq 0.0002 z \text{ for } i, j \in \{x, y, z\}.$$

We know that the maximum distance of the catheter from the

EM base unit is 300 mm. Clearly, the worst case distortion at this distance (9 mm) is too great for us to achieve our desired accuracy. Calibration is required.

For the purposes of calibration, an optical tracking system is available. The tracker itself is mounted on a tripod that is not completely stable. However it is able to simultaneously measure the positions of many optical markers (up to about 30) to very high accuracy (0.1 mm). A supply of optical markers with stick-on backs is available. A number of these have been attached at **unknown** positions around the peripheral top surface of the EM tracker base unit, which is 500 mm square. In addition, a calibration object has been constructed from a small square of thin flat plastic material (approximately 50 mm on a side). A single 6 DoF EM marker and a number of optical markers have been glued to this square at **unknown** positions. The optical markers are approximately at the corners of the square and the EM marker is approximately in the middle. The translational accuracy of the 6 DoF EM marker is the same as that of the 3 DoF marker. The orientation accuracy, however, is uncannily accurate near the base unit. If the actual pose of the EM marker is  $\mathbf{F}_C = [\mathbf{R}_C, \mathbf{p}_C]$ , then the measurement error will be

$$\|\Delta \mathbf{R}_C\| \leq 0.00005 z \text{ radians where } z \text{ is the distance in mm above the EM tracker base unit.}$$

Assume that you have a computer workstation that is able to simultaneously query the positions of all the optical markers and the spare EM marker. At each time frame  $t$ , the system is able to measure the following:

$\vec{\mathbf{a}}_i(t)$  = the measured positions of the optical markers on the calibration object relative to the optical tracker

$\vec{\mathbf{b}}_j(t)$  = the measured positions of the optical markers on the EM tracker base relative to the optical tracker

$\mathbf{F}_C(t)$  = the measured position and orientation of the 6 DoF EM marker relative to the EM base unit

Also, you may assume that there is a very inexpensive plastic robot available that can move the plastic calibration object around in the workspace. The robot itself is very inaccurate (errors on the order of 5 mm position and 5 degrees orientation) but can be used to automate the calibration procedure.

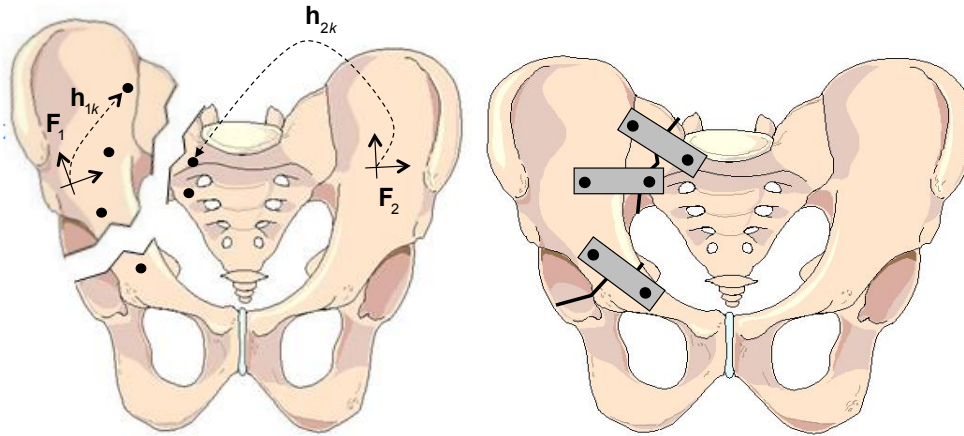
- A. Describe a procedure for defining coordinate systems  $\mathbf{F}_A(t)$  and  $\mathbf{F}_B(t)$  and associated offset vectors  $\vec{\mathbf{a}}_i^{offset}$  and  $\vec{\mathbf{b}}_j^{offset}$  with the optical markers attached to the calibration object and EM tracker base, respectively, so that  $\vec{\mathbf{a}}_i(t) = \mathbf{F}_A(T) \cdot \vec{\mathbf{a}}_i^{offset}$  and  $\vec{\mathbf{b}}_j(t) = \mathbf{F}_B(T) \cdot \vec{\mathbf{b}}_j^{offset}$
- B. Let  $\mathbf{F}_G$  be the unknown transformation between EM tracker and  $\mathbf{F}_B(t)$  coordinates, so that a point at a location  $\vec{\mathbf{v}}$  relative to the EM tracker base coordinate system will be at position  $\mathbf{F}_G \vec{\mathbf{v}}$  relative to  $\mathbf{F}_B(t)$ . Let  $\mathbf{F}_H$  be the unknown position of the calibration object EM marker relative to the  $\mathbf{F}_A(t)$  coordinate system. Give an expression for  $\mathbf{F}_H$  in terms of  $\mathbf{F}_G$ ,  $\mathbf{F}_A(t)$ ,  $\mathbf{F}_B(t)$ , and  $\mathbf{F}_C(t)$ .
- C. Give a procedure for determining  $\mathbf{F}_G$  and  $\mathbf{F}_H$  to a positional accuracy of about 0.5 mm. This procedure will doubtless include placing the calibration object at multiple positions and (maybe) orientations relative to the EM tracker base and acquiring data. Your answer should include a list of (approximate) poses where the object is to be placed for the purpose of this calibration. You should also give sufficiently detailed formulas and computational procedures so that it is clear how your method would be implemented.

D. Assume now that your calibration method has been wildly successful, and that  $\mathbf{F}_G$  and  $\mathbf{F}_H$  are known, with negligible orientation error and positional error on the order of 0.2 mm. Assume that you plan to use the following calibration scheme:

- Place the calibration object in a sequence of positions about the work volume so that the calibration EM marker is placed in a series of known positions  $\vec{\mathbf{v}}_{ijk}(t)$  arranged roughly in a volumetric grid, where the grid spacing is roughly  $D$  mm apart relative to the EM tracker base. I.e., if  $\vec{\mathbf{v}}_{ijk} = [x, y, z]$ , then  $\vec{\mathbf{v}}_{i+1, j, k} = [x + D, y, z]$ , etc. Measure the positions  $\vec{\mathbf{p}}_C(t)$  of the EM marker at each point.
- Given an arbitrary measured position  $\vec{\mathbf{v}}_{meas} = [x_{meas}, y_{meas}, z_{meas}]$ , determine the corresponding calibrated position  $\vec{\mathbf{v}}_{calib}$  by trilinear interpolation.

Give some bounds on how accurate your value for  $\vec{\mathbf{v}}_{calib}$  is likely to be for a point with  $z_{meas}$  no further than 300 mm from the tracker. What value of  $D$  will be required to guarantee that the error in the corrected location of the catheter EM marker is less than 2 mm? **Hint:** Note that I'm looking for an approximate and conservative bound on the error value. You can make any reasonable simplifying assumptions, so long as you say what they are.

## Question 2



**Figure 2**

Consider the situation shown in Figure 2 (left). The patient has a severely broken bone (here, a pelvis). The goal is to reduce the fracture (i.e., realign the bone fragments) and bolt them together using screws and plates as shown in Figure 2 (right). The normal procedure goes roughly like this:

1. Reduce the fracture.
2. Align the plates with the bone fragments.
3. Drill pilot holes using the plates as a drill template.
4. Insert bone screws through the plates to secure the bones together.

However, we will here consider an alternative procedure, based on CT-based planning. The preoperative sequence is roughly as follows:

1. Obtain a CT scan of the patient, and segment the scan to identify the bone fragments. Let  $\mathbf{F}_1$  and  $\mathbf{F}_2$  be the coordinate systems associated with each bone fragment
2. In the planning system, determine how bone fragment 1 needs to be repositioned relative to bone fragment 2 so that the fracture is reduced. I.e., determine a coordinate transformation  $\mathbf{F}_{red}$  such that  $\mathbf{F}_1 = \mathbf{F}_{red} \mathbf{F}_2$  when the fracture is reduced.
3. Determine the design of plates to hold the bone fragments in this relative pose. Determine the locations  $\vec{h}_{1k}$  and  $\vec{h}_{2k}$  relative to  $\mathbf{F}_1$  and  $\mathbf{F}_2$  of holes to be pre-drilled into the bones to hold the plates (and, hence, the bone fragments) in the right pose.
4. Prefabricate the plates.

In the operating room, the bone fragments are to be registered to the preoperative plan, the holes are to be drilled in the desired places, and the plates are to be attached to reduce the fracture. For the purpose of this exercise, we may assume that the procedure will be successful if the holes are drilled within approximately 2 mm of the planned locations, though higher accuracy may in some cases be desirable.

Your problem is to design a system to perform the intraoperative portion of this procedure. Very broadly, the steps are:

1. Set up the needed equipment and perform any needed calibrations
2. Set up the patient (e.g., put the patient into a fixture, attach reference markers, etc.)
3. Perform the registration of the intraoperative system to the preoperative plan
4. Perform the drilling of the holes (or assist the surgeon in drilling them)
5. Attach the plates & screws

Each of these steps may be performed in one of several ways, depending on the equipment and algorithmic approach chosen. You should explain your choices of equipment (navigation system, robot, trained termites, or whatever) and approach clearly and justify it relative to possible alternatives. What are the advantages? What are costs/disadvantages?

For your chosen approach, you should provide sufficient additional detail, formulas, etc., so that one of ordinary skill in the art can be expected to implement it. You do not need to recite all the steps of known algorithms in great detail, but you should provide a pointer that identifies the algorithm, together with descriptions of the inputs and outputs, making clear where the input data is coming from. One or two block diagrams or sketches may prove useful. Likewise, it is really important to make clear what all the relevant coordinate system relationships are and address accuracy issues to the extent practical.

Again, there is no one right answer to this problem. Also, do not write an extensive book. Try for reasonable succinctness roughly comparable to what you did in HW#1.